LEARNING GEOMETRY ONLINE: A CREATIVE INDIVIDUAL LEARNING EXPERIENCE

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—Abstract—

The increasing influence of technology in education has made us want to understand how teachers will view its use when teaching Euclidean geometry. Teachers’ technological knowledge, skills, competence and readiness are crucial, in their preparation to integrate technologies in their future classrooms. This paper is part of a larger study that looked into the experiences of student teachers when using a form of online technology, which utilised meaningful interactions to teach and learn high school geometry. The study was conducted with ten mathematics student teachers at a South African university using a mix methods approach using worksheets, interviews, surveys and observations. This case study sought to capture the students’ knowledge of technologies when teaching circle geometry through a careful implementation of Geometric Habits of Mind and an Instructional Design model. The intention of the support programme was that students will reproduce their learning experience, during this study, in their classrooms. According to results the perceptions of technology can be categorised as: Knowing the essentials, A tool in learning geometry and A catalyst to learning geometry. It can be clearly stated that all of students liked the readily available learning tool online.

Key Words: online, interactive, dynamic geometry, visual, ICT

JEL Classification: I2, I23, Q5, Q55

1. INTRODUCTION

The KwaZulu-Natal (KZN) educational summit 2015 theme was: All doors of learning and culture shall be opened. This was adopted from the South African Freedom Charter that was drafted more than 50 years ago. This theme underpinned the integration of ICTs (Information Communication Technologies) into schools, thus becoming the focus of the summit. Technology has become the building block of our society with the rise of the fourth industrial revolution. Globalisation of technology enhances international as well as local skills, knowledge, competition and development. ICTs such as electronic communication, wireless devices, the internet, smartphones and PC tablets are used on a daily basis. There is a rapid acceleration of technological advancement that is continually creating platforms for new thoughts and creative ideas. This is giving rise to new benchmarks in industry. However, all this starts from the learning institutions, where we are able to guide students to train their thoughts and nurture these creative patterns into dynamic ideas. Teacher education institutions throughout the world have adopted educational reforms in response to this changing landscape (Ibrahim, & Callaway, 2018; Leendertz, Blignaut, Nieuwoudt, Els & Ellis, 2013).

This increasing influence of technology in education has made us want to understand how South African pre-service teachers (PSTs) will view its use when teaching Euclidean geometry- a topic that was once omitted from the Mathematics curriculum before the Curriculum and Assessment Policy Statement (CAPS) reinstated it in 2012. In this study, views of PSTs on the use of technology in teaching geometry was taken as subjective knowledge. The topics that were chosen are Theorem of Pythagoras, which is taught in the GET phase, and circle geometry which is taught in the FET phase in line with the South African curriculum, the Curriculum Assessment Policy Statement (CAPS).

Pre-service teachers’ technological knowledge, skills, competence and readiness are crucial in their preparation to integrate technologies in their future classrooms. These student teachers must have the aptitude and talent to see possible solutions that go beyond the normal thought patterns. The study focuses on Dynamic Online Environments (DOEs) - the term assigned to the type of technology utilised. For the purpose of the study, DOEs are virtual spaces on the Internet which can be used for teaching and learning of geometry. The online platform has made information easily accessible anywhere in the world via multiple computing devices. This online mode of learning is referred to as m-learning (mobile learning) (Annan, Ofori-Dwumfuo, & Falch, 2018; Alrasheedi & Capretz, 2015). Many institutions now offer all lessons and activities online (Eady, Jones, Alony, & Berry, 2018), which
promotes environmental sustainability and offers learners the flexibility of distance learning. This paper focuses on the following research question flowing from the aforementioned narrative: How do pre-service teachers view the use of DOE technology based teaching methods to learn Euclidean geometry?

2. PARTICIPANTS AND CONTEXT

A case study methodology was employed to conduct this study since Thomas (2011) notes that the aim of such a study is to provide an in-depth description of a particular event or scenario. This study was structured in a natural setting for PSTs to use their understanding to make sense of or interpret the given phenomenon. The site chosen for the study was a university in KwaZulu-Natal, South Africa. At this university, PSTs have the option to register either for a Bachelor’s Degree or Certificate in Education. This university’s LAN was adequate to commence data collection as it offered well-equipped and updated computer technologies with Internet access. In addition, it was convenient for the participants as they were familiar with the campus and surroundings. The selection criteria were based on quota sampling. The mathematics student teachers had to meet the following criteria: 1. Registered in the School of Mathematics Education, 2. Phase specialisation in high school (upper senior FET or FET band), 3. In their 2nd, 3rd or 4th year of study. The invitation was sent out via e-mail, twenty-six students accepted based on the criteria but ten participated in the study. The remaining sixteen did not attend due to unforeseen circumstances.

Two geometry topics from Euclidean geometry were selected, the Pythagorean Theorem encountered in the GET (General and Education Training) phase at high school and circle geometry encountered in the FET (Further Education and Training) phase at high school. For the purpose of this paper, the technology used will be examined through the lens of circle geometry only. Circle geometry is a topic which is introduced in Grade 11. This topic falls within Euclidean geometry, which was re-introduced into the South African syllabus in the FET phase with the implementation of the CAPS in 2012. Most of the PSTs in this study would not have studied circle geometry in the FET phase, as they completed Grade 12 prior to the implementation of the CAPS, or they would have stopped teaching it from 2008 to 2012. Thus it was interesting to determine how PSTs would view teaching circle geometry using technology.

Four circle geometry theorems were selected for PSTs to engage with. The selection was based on the appearance and occurrence of the theorems directly and indirectly, in the Mathematics NSC (National Senior Certificate) examinations from 2007 to 2013. The examiners’ reports were also analysed in order to detect problem areas
and performance levels in circle geometry. The first theorem selected was: *if a line is drawn from the centre of a circle perpendicular to a chord, then it bisects the chord* otherwise known as *perpendicular from centre to chord*. The proving and application of this theorem appeared in the 2013 NSC Mathematics paper 3. The next theorem selected was: *If an arc subtends an angle at the centre of a circle and at any point on the circumference, then the angle at the centre is twice the measure of the angle at the circumference*, otherwise known as *angle at centre is twice the angle at circumference*. The proving of this question appeared in the 2009 NSC Mathematics paper 3. The third theorem selected was: *If a quadrilateral is cyclic, then the opposite angles are supplementary*, otherwise known as *opposite angles of cyclic quad are supplementary*. The proving of this theorem was in the last NSC Mathematics HG paper 2, in 2007. The final theorem selected was: *the angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment*, otherwise known as *tangent-chord theorem*. This theorem was in the NSC Mathematics paper 3 in 2010 and 2011. Application questions based on this theorem can be found in all Mathematics examinations papers from 2007 to 2013.

Prior to the scrapping of the Mathematics Paper 3 in 2013, the Subject Report from Universities suggested that learners who wanted to pursue a career in Engineering should consider taking this paper (Department of Basic Education, 2011b). Pillay and De Villiers (2013) noted that “university lecturers in mathematics, science and engineering see geometry as an integral part of mathematics, and teaching it at school is important for university preparation” (p. 36). On closer examination of Geometry outside the walls of the classroom, we can see that Geometry is a critical component in many fields of study like Engineering, Cartography and Architecture.

Currently South Africa is experiencing the unfolding of the 4th industrial revolution which includes the unveiling of artificial intelligence, big data, robotics and other related developments. In readiness to “ensure that South Africa effectively embraces the 4th Industrial Revolution, the President has appointed a task team to focus on the 4th Industrial Revolution” (SONA, 2019). Thus as society begins to change there is a need to explore new avenues that make learning relevant. In terms of teaching, technological knowledge includes the ability to use computers, the Internet or digital material to teach learners (Leendertz, Blignaut, Nieuwoudt, Els & Ellis, 2013). Technology can be in a hardware (device) or software (program) form that makes one’s life easier. There have been impressive advances in teaching technologies over the years ranging from overhead projectors to PC tablets in the classroom. Educational technologies are constantly and rapidly being updated and have become so accessible that it is becomes non-negotiable to integrate into
classrooms. Educational technologies enhance and create a captivating learning environment (Squires, Turner, Bassendowski, Wilson & Bens, 2017). Using technologies in education allows for engagement, interest and interaction which leads to the development of learners’ thinking, as compared to traditional teaching practices such as ‘chalk and talk’ (Chigona, Chigona, Kayongo & Kausa, 2010). It is important to note, that the use of technology does not necessarily mean that it is being used critically and meaningfully in the classroom. One can easily fall short in demonstrating the use of technology to the extent that it is no different to traditional teaching methods (Naidoo & Govender, 2014). Therefore strategic planning is essential for technology to be successfully adopted for harmonious learning experience.

3. THEORETICAL FRAMEWORK

For the purpose of this study, technology is defined as something invented that is useful to solve a problem. With this in mind, two theoretical frameworks underpinned this study: Geometric Habits of Mind (GHOM) and Instructional Design (ID).

3.1. Geometric Habits of Mind

Habits of mind are productive approaches that promote reasoning and thus learning (Köse & Tanişli, 2012). The development of one’s Habit of Mind establishes an in-depth rather than a superficial understanding of the subject content. These habits are normally applied to algebra and geometry and are termed Algebraic Habits of Mind (AHOM) or Geometry Habits of Mind (GHOM). This study drew on the work of Driscoll (2001) and Driscoll, DiMatteo, Nikula, Egan and Mark (2007), that describes the following four GHOM (Figure 1) that contribute to an understanding of Euclidean geometry:
### 3.2. Instructional design

Instructional Design (ID) is a collection of theories that foster learning through the application of an instructional method (Paquette, 2014). The application is a process that facilitates strategic planning of learning activities, thus bringing coordination. Instructional design can take the form of any type of multimedia such as web pages with photos, text, and video streaming, etc. This exposes the learner to multiple perspectives when solving a problem. According to Paquette (2014), ICT-based tools and methods are crucial for instructional designers in planning the delivery of learning systems. To promote proper integration of technology in geometry, this study adapted the following instructional design principles by Mark, Cuoco, Goldenberg and Sword (2010) in alignment with GHOM to create instructional materials (DOEs):

#### Figure-2: Instructional design principles

<table>
<thead>
<tr>
<th>Smooth integration</th>
</tr>
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<tbody>
<tr>
<td>Experience experimentation</td>
</tr>
<tr>
<td>Low threshold-high ceiling approach</td>
</tr>
<tr>
<td>Being organised</td>
</tr>
</tbody>
</table>

3.3. The link

In this study the smooth integration principle overarches the entire operation of adapting technology meaningfully into the Mathematic topic (Geometry). The link of the ID principles and GHOM are shown in Table 1.

Table 1: Linking of GHOM and ID

<table>
<thead>
<tr>
<th>ID</th>
<th>GHOM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experience experimentation:</td>
<td>Reasoning with relationships, Investigating Invariants and Balancing Exploration and Reflection.</td>
</tr>
<tr>
<td>learning must allow for trial and error exploration of abstract concepts</td>
<td></td>
</tr>
<tr>
<td>Low threshold-high ceiling approach:</td>
<td>Reasoning with relationships, Generalising geometric Ideas and Investigating Invariants.</td>
</tr>
<tr>
<td>the material becomes progressively more difficult, offering a challenge.</td>
<td></td>
</tr>
<tr>
<td>Being organized:</td>
<td>Generalising geometric Ideas and Balancing Exploration and Reflection.</td>
</tr>
<tr>
<td>it is important that bits of mathematical concepts be meaningfully linked, to show the interconnected network – the use of a prior theorem when proving a newly introduced theorem.</td>
<td></td>
</tr>
</tbody>
</table>

4. DESIGNING THE ONLINE ENVIRONMENT

This study utilised dynamic software (Geogebra) as a research tool, as it is a powerful and versatile tool that offers learners an optimal learning environment. For the purpose of this study, dynamic software refers to dynamic geometry application software such as Geometer’s Sketchpad, Cabri, Logo, Geometric Supposer, Geogebra and Wolfram’s Mathematica, to name but a few. The key property of dynamic software is that it offers the learner freedom to move shapes and graphs, as compared to the static convention of sketches. It enables instantaneous movement of an object such as a parabola graph as the values change; a, b and c of \( f(x) = ax^2 + bx + c \) for example. Take, for example, a part of the DOE for the theorem that states (Figure 3), *if an arc subtends an angle at the centre of a circle and at any point on the circumference, then the angle at the centre is twice the measure of the angle at the circumference otherwise known as angle at the centre is twice the angle at circumference*. This theorem is represented in three
different ways in Figure 3\textsuperscript{1} below, with Figure 3 “\textit{Figure 1}” being the most basic representation. The click and drag action can be used such that Figure 3: “\textit{Figure 1}” can be manipulated to identically represent Figure 3: “\textit{Figure 2}” and Figure 3: “\textit{Figure 3}”. In addition, one can notice the instantaneous change in the size of the angles. The use of educational technologies such as dynamic software offers great opportunities for an exploratory style of teaching and learning of mathematics (Karataş & Güven, 2015; Mavani & Mavani, 2014 and Hölzl & Schäfer, 2013). However, the implementation of technology does not guarantee effective teaching and learning. As Davies (2013) remarks, the use of technology does not guarantee meaningful learning; neither does it mean that it is utilised in a meaningful way.

\textbf{Figure-3: Part of the DOE for angle at centre is twice the angle at circumference}

Source: http://fibonacci.africa/CircleGeoT3/

Exposure to educational software helps to make abstract concepts simple and tangible which results in knowledge fitting easily with a learner’s current knowledge structure. For the purpose of this study, dynamic content was defined as the user’s freedom to make instantaneous changes to an object’s properties. Learners and teachers can manipulate the constructions made through the software dynamically, as the software creates an environment which one can explore. Accompanying technology based learning are tips or hints which guide and probe one’s understanding. These provide assistance in solving the problem and

\textsuperscript{1} Figure 3 has three sub figures Figure 1, Figure 2 and Figure 3 which is built into the design of the applet. Reference to the subfigures are made in quotations and italics
developing understanding which provides scaffolding. The Cognition and Technology Group at Vanderbilt (1997) termed these hints *embedded data*. This relates directly to the Zone of Proximal Development (ZPD) proposed by Vygotsky (1978). When one reaches a dead end or mental block, one can seek help from the embedded data. The embedded data/ hints draw on one’s current understanding, taking one from the known to the unknown. These hints or data play a crucial role when instructional time is limited, allowing an individual to cover more work. Geometry is allocated a limited time frame in Grades 10, 11 and 12 (Department of Education, 2011a). Thus, the embedded data or hints are useful in the learning process, bearing in mind the number of teaching and learning disturbances such as sports, tours, public holidays and unforeseen events.

5. DATA COLLECTION

A mixed methods approach enabled for the construction of detailed descriptions and more complete explanations. Data was collected from PSTs by administering questionnaires in the form of worksheets, interviews, observations and surveys that required responses in the form of a Likert scale. These instruments included open- and closed-ended questions. The data collection process took the form of a workshop, which unpacked the relevant circle geometry theorems prescribed by the CAPS document through the use of a chosen technology.

6. DISCUSSION AND ANALYSIS

The data analysis revealed a range of factors that contributed to PSTs' views on the use of technology to teach Euclidean geometry. These include their attitudes and beliefs about technology and their ability to use it effectively in teaching. The observations and focus group interviews generated qualitative data, which were analysed holistically with the pre-questionnaire, worksheet based tasks and post-questionnaire to identify the emerging themes. This in-depth analysis coupled with thorough inspection identified consistencies and inconsistencies in the findings, which sought to provide responses to the question outlined in the study. Engaging with all collected data by means of coding and categorisation led to the identification of common themes. The following six themes emerged: motivation through the use of technology, visualisation is key when teaching geometry, using technology to teach is not for everyone, Geometrical conceptual growth through technology, the use of technology promotes independent thinking and GHOM, and the quality of teacher training at university.

This paper will focus on the key findings based on the perceptions of technology in Geometry: Knowing the essentials, A tool in learning geometry and A catalyst to
learning geometry. The integration of technology must be thoroughly understood in order to harmoniously blend technological elements into the classroom. In general, the use of technology tools depends on the teacher’s attitude, as teachers are the driving force in the learning environment. As remarked by the President of South Africa in the State Of the Nation Address “we are expanding the training of both educators and learners to respond to emerging technologies including the internet of things, robotics and artificial intelligence” (SONA, 2019: 25). The study revealed three categories of views on the integration of technology in the classroom from a geometric perspective. Figure 4 below shows the views on technology in geometry.

Figure 4: The role of technology in Geometry

6.1. Knowing the essentials

Using technology in a fairly harmonious manner but not necessarily on a daily basis, would characterize one as being technology literate. The acquisition of basic computer principles and word processing skills forms the foundation for being computer literate as stipulated in the White paper on e-Education (Department of Basic Education, 2004). Basic use of a computer which one might encounter during lesson preparation like switching it on, navigating through windows, opening files, editing files, saving, printing, downloading from the Internet and so forth. This would describe one as being computer literate. The responses to the statement: I know how to use a computer averaged 4.6 on the Likert scale and a 1.6 average was found in response to I was frustrated sitting alone in front of a computer. Thus, it can be assumed that the majority of the participants have basic skills in using a computer and can be deemed computer literate. Further it was noted that Computer
Literacy is a compulsory module in the PSTs’ first year of study, and e-communication was used widely at the university. This could improve the efficiency of school administrative functions such as storing data electronically, in turn decreasing the time spent on administrative functions (Chigona, Chigona, Kayongo & Kausa, 2010) when PSTs are at school and later graduate. This enables teachers to spend more time on lesson planning and other activities. However, teachers that are computer literate cannot design or construct their own geometry content but merely utilise available sources. An average score of 3, meaning a neutral response, was obtained for the statement - *DOE should only be used at university level for teacher training*, while an average of 3.4 was obtained for *DOE is too technical to use at school*, suggesting that some sort of specialised knowledge is required.

6.2. A tool in learning geometry

According to the Department of Basic Education (2004), the White Paper on e-Education notes that every teacher should not only have access to but the means to obtain a personal computer for personal use, for preparation of lessons and administration. In the following statements participants averaged 1.4 on the Likert scale: *I dislike the use of technology when teaching mathematics* and 1.6: *I have no intention to use DOE in my future mathematics lessons*. This implies that technology is likely to be used to teach mathematics, let alone geometry. In addition, the majority of the participants agreed with the following statements: *The DOE is simple and understandable to use* (4.3 average), *DOE increases efficiency of Mathematics Education* (4.7 average) and *Dynamic software can improve mathematics results* (4.4 average). Therefore, technology like DOE is seen as a valuable teaching tool in geometry in order to improve one’s understanding. An analogy to *technology as a geometry tool*, is the use of a hammer to knock a nail into wood. The tool which is the hammer refers to the chosen technology, for example, PowerPoint presentations, an overhead projector, mind map software, clickers, dynamic software etc. The wood refers to the geometry subject content and the nail refers to the learner who pierces into the geometry subject content. This simply means that technology refers to the hammer being used to accomplish the task at hand, that is, to teach the learners. It is in this category that one uses technology as a tool in the geometry teaching process, similar to using a ruler to draw a straight line and nothing more. Govender (2012), pointed out by that the mere use of computers is inadequate for achieving educational change.
6.3. A catalyst to learning geometry

The integration of technology should not focus exclusively on computer skills or the utilisation of technology to complete a geometry task. Instead, it should be used more meaningfully within the context of the teacher’s beliefs, which include how the teacher perceives effective teaching and how technology can alter the traditional roles of teachers and learners. Since there are greater efforts to prepare South Africa for the digital age, as mentioned in the SONA (2019; 24) that “already, 90% of textbooks in high enrolment subjects across all grades and all workbooks have been digitized”. The teacher should demonstrate more than being technologically literate when using technology to enhance a geometry lesson. The majority of the participants agreed with the following statements: *DOE increases the quality of Education* (4.4 average) and *I would like to learn more about dynamic software in mathematics* (4.7 average). It can thus be concluded that PSTs regard technology as a valuable aid in learning geometry, rather than as simply being used to transfer knowledge/a concept. The hints in the DOEs in this study act as a gateway when one hits a mental block. Polya (1957) recommends that each problem should be accompanied by a hint that will lead to the answer. The hints are a form of questioning of one’s mental progression, which allows for self-correcting before intervention by the teacher. Naidoo and Govender (2014), mention that placing an educational concept online blended with an appropriate tool results in self-regulation (Figure 5).

**Figure 5: The effects of online technology on teaching and learning**

![Diagram showing the effects of online technology on teaching and learning]

*Source:* Naidoo and Govender: 2014: 2

The role of technology, commonly computers, in the teaching and learning of mathematics is becoming increasingly important, as it lays the foundation for the advancement of mathematics (Tatar, 2013). The adoption of technology should be driven by the desire to bring about change in the learning environment by
integrating the chosen technology meaningfully in the classroom. It is important to note that the teacher should use technology to enhance the geometry learning process, rather than learning how to use the technology.

7. CONCLUSION

The use of technology as a catalytic tool to learning such as DOE in this study making geometry content interactive and readily available can have a positive effect. Meaningful integration of technology has the possibility to change and enhance understanding. DOE gives learners and teachers the freedom to discover concepts and cause-effect relationships through exploration and experimentation. However, mere implementation of technology does not guarantee effective learning. Derived from the themes and the perspectives of technology, effective learning occurs when one interacts with the objects under study with the proper use of technology, by promoting geometric thinking through the development of GHOM. Developing learners’ GHOM should be critical to the teacher as it prepares them to solve and make sense of complex geometric problems. GHOM can be achieved through the implementation of technology like DOEs which give instructions and feedback during one’s interaction with the geometry concepts at hand. Drawing from the larger study the prospective teachers’ negative conceptions of geometry and their lack of confidence are altered when they realise the learning potential under the guidance of technology. It is suggested that, wherever possible, teachers make use of visual and virtual reality technology, thus creating visual representation and promoting physical movement during the teaching of geometry. This improves understanding of geometrical relationships and properties more than presenting information in a verbal and static 2-D form.

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