IS IT POSSIBLE TO REDUCE THE STOCK OF FOREIGN RESERVES?

Marcello Spanò  
Dipartimento di Economia, Università dell’Insubria  
Via Monte generoso, 71 – 21100 Varese – Italy  
E-mail: Marcello.spano@uninsubria.it

—Abstract—

This work determines the optimal reserves to short term debt ratio of an exporting economy indebted in foreign currency and suggests possible remedies to reduce it. Theoretical results and numerical simulations establish that the ratios recently observed reflect the increasing weight assigned to the risk of firms going bankrupt. They also establish that neither a lower risk premium charged by international lenders nor a lower exchange rate volatility reduce the stock of reserves significantly. Full elimination of the need to hold reserves to prevent financial crises should rely either on limiting foreign capital inflow or on reforming the international monetary system.

Key Words: reserves, short term debt, exchange rate, financial distress, bankruptcy, hedging.  
JEL Classification: F32; F34; F41.

1. INTRODUCTION

This study contributes to discussion on the causes of rising foreign reserves and on possible reforms aiming at reducing the optimal reserves to short term debt ratio in the emerging economies.

Since the 1990s, and increasingly after the 1997-1998 crisis in East Asia, emerging economies have changed their attitude towards international reserves and raised hoarding in a quantitatively impressive way. (Aizenman and Lee, 2007; Devereux and Sutherland, 2009; Rodrik, 2006). Recent theoretical studies (the precautionary – or self-insurance – view) argue that hoarding international
reserves allows emerging economies to cushion the real output of the economy from a slowdown in capital inflow. (Calvo, 1998; Feldstein, 1999; Bordo et al., 2010) Most of them focus on output stabilisation and build on the assumption that a sudden stop (of capital inflow) is an uncertain event occurring with a given probability. (Jeanne and Ranciere, 2006; Jeanne, 2007; Aizenman and Lee, 2007; Cheung and Qian, 2007; Garcia and Soto, 2004). However, these studies do not explicitly consider the impact of a capital reversal on the exchange rate, all the variables being denominated in the same currency.

The model presented in this paper contributes to overcoming this limitation by building on the assumption that the exchange rate is uncertain. It focuses on the immediate and most important consequence of capital outflow, which is precisely the explosion, following the depreciation of domestic currency, of the value of both capital and interest of the existing short term debt denominated in foreign currency (typically, US dollars). The model is simulated by capturing the documented spread between the low yield on liquid reserve assets and the cost of external borrowing (Baker and Walentin, 2001; Rodrik, 2006; Stiglitz, 2006). The rationale for holding reserves, in this model, is to generate financial income which is negatively related to short term external debt burden, thus to financial distress. Financial crises are modelled as the bankruptcy of a representative firm with foreign short term liabilities, generated by a sharp depreciation of the domestic currency. However, reserves also allow for hedging against exchange rate variations that are not generated by extreme events.

2. THE GENERAL MODEL

The model describes a decentralised small open economy that produces and consumes a single perishable good and lasts for two periods, \( t \) and \( t + 1 \).

A representative firm carries out a new investment in period \( t \), \( I_t \), raising financial funds either from foreign lenders (\( D_t \)) or from domestic owners (\( V_t \)). The firm runs the risk of going bankrupt, depending on the realisation of the shock to the exchange rate, \( R_{t+1} \), which is the only unknown variable determined between time \( t \) and time \( t + 1 \). Given the rate of exchange for the domestic currency against the foreign currency in the first period, \( \varepsilon_t \), we assume that foreign exchange in the second period is \( \varepsilon_{t+1} = R_{t+1} \varepsilon_t \), with \( R_{t+1} > 0 \) and \( E[R_{t+1}] = 1 \).
$D_t$ is one period claims from foreigners on the country, it is denominated in foreign currency (US dollar) and is given (capital inflows entirely depend on supply from foreign creditors). The budget constraint of the new investment at time $t$ is thus given by (1).

Period $t+1$'s output from the new investment is given by $\theta_{t+1}f(I_t)$, where $f(I_t)$ is a strictly increasing and strictly concave investment function, and $\theta_{t+1}$ is a multiplicative shock depending upon the exchange rate, given by (2), where $\alpha$ can be interpreted as a measure of the degree of dependence of domestic production over foreign demand. For the purpose of describing an economy that exports its product abroad, it is reasonable to consider non negative values only, thus $0 \leq \alpha \leq 1$.

The firm goes bankrupt when the net revenue from the sale of its product is smaller than foreign debt service (principal and interest), that is, when the exchange rate depreciates over a threshold, $\rho_{t+1}$, given by (3), where $\rho_{t+1}$ denote the interest rate associated to foreign debt. We assume that both the numerator and the denominator of (3) take positive values, in order to have financial distress (probability of going bankrupt) associated to exchange rate depreciation and revenues from the investment not lower than the funds provided domestically.

Let $G_t(\rho_{t+1}) > 0$ be the conditional distribution, with density $g_t(\cdot)$, of the shock to the exchange rate, $\rho_{t+1}$, given the information available at time $t$. The probability of going bankrupt is thus given by $1 - G_t(\rho_{t+1})$. We assume that the payout, in case of bankruptcy, is a proportion $\beta$ of the level of debt raised at time $t$ expressed in local currency, i.e. $B_t = \beta_0 D_t$. The expected cost of foreign debt, in local currency, is given by (4).

The expected profit function can be written as the difference between the expected net revenues on investment and the expected cost of debt: $E_t[I_t] = f(I_t) - V_t$. The firm sets the value of $I_t$ according to the first order condition (5). In order to describe a realistic economy where a rising level of investment, *ceteris paribus*, is associated to a rising probability of going bankrupt, we assume that the sensitivity of the bankruptcy threshold to the level of investment is a decreasing function of the investment, i.e. $\frac{\partial \rho_{t+1}}{\partial I_t} < 0$. 

195
The representative agent is endowed with the income $Y$ in both first and second periods. To finance future consumption, the agent can transfer wealth over time through the new investment's equity holding, $L_e$, and international currency reserves, $F_e$. Foreign currency reserves are one period claims of the country on foreigners, denominated in the same currency as the firm's foreign debt. We assume that the interest rates associated to reserves, $r_F$, is given and that $r_L > r_F$. At the beginning of period $t$ and at the end of period $t+1$, the country neither holds uncollected claims on foreigners, nor do foreigners hold uncollected claims on the country. In case of bankruptcy, the quote of the debt service which is not paid by the firm will be paid by the representative agent.

The representative agent maximise the time- and state-separable discounted utility function (6) of the aggregate consumption, $C$, under the first and second period budget constraints, i.e. respectively (7) and (8), where $\beta$ is the representative agent's intertemporal discount factor, $\omega(.)$ is strictly increasing and strictly concave, total assets are $\alpha_e = L_e + e_t F_e$, and assets' net income is the sum of the income from equities and reserves, as in equation (9).

Given the firm's decision on the optimal level of investment, $L_e$, assuming that, in equilibrium, equity market clears (i.e. $L_e = V_e$), the representative individual’s first order condition (intertemporal Euler equation) is given by (10).

\begin{align*}
L_e &= V_e + e_t D_e \tag{1} \\
\theta_{t+1} &= \alpha(\mu_{t+1} - 1) + 1 \tag{2} \\
\mu_{t+1} > (1-a)^f(\mu_t - c_t) \tag{3} \\
K_e &= e_t D_e \left[ (1+r_c) \int_0^{\bar{\mu}_{t+1}} \mu_{t+1} g(\mu_{t+1}) d\mu_{t+1} + b \left[ \int_{\bar{\mu}_{t+1}}^{\infty} g(\mu_{t+1}) d\mu_{t+1} \right] \right] \tag{4} \\
f^T(L_e) &= 1 + e_t D_e g(\bar{\mu}_{t+1}) \frac{\partial f}{\partial \mu} [(1+r_c)\bar{\mu}_{t+1} - b] \tag{5} \\
u &= u(C_e) + \beta E_e u(C_{e+1}) \tag{6}
\end{align*}
3. SIMULATION SOLUTION

To find numerical solution, we assume that the period utility function is a standard constant-relative-risk-aversion utility function, \( u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \), with \( \sigma > 0 \); the shock to the exchange rate, \( \mu_{t+1} \), is gamma-distributed with shape \( k \) and scale \( \frac{1}{k} \) (mean value is \( k^{1/k} = 1 \); the investment function is \( I_t = w + \frac{s(1-w)^{s-\sigma}}{(1-\sigma)} \), with \( 0 < \sigma < 1, s > 0, w > 0 \), where \( s \) is a scale parameter, \( w \) represents a fixed part of the investment below which the return is zero, and \( (I_t - w) \) is the variable part of the investment whose return behaves as a Cobb-Douglas normalised with respect to the labour factor.

To help discussion, we define four new variables and simulate their value: (i) the gross national product, GNP, at time \( t+1 \), derived by summing the domestically produced income, GDP, and the value of the interest and capital gain/losses on prior net foreign position (to evaluate the total impact of the shock to the exchange rate on the income available to the economy), given by (11); (ii) the sensitivity of total income to the shock to the exchange rate, \( \gamma \) (to quantify the additional resources generated by any change in the foreign exchange rate), given by (12); (iii) the "neutral" ratio \( \frac{F_t}{D_t} \), or Greenspan-Guidotti ratio (the ratio that would fully stabilise the total income of the economy with respect to the shock to the exchange rate), given by (13); (iv) the level of debt implying no bankruptcy, \( ndD \) (the level of debt such that the debt service is perfectly offset by the change in revenues from the investment, thus bankruptcy is ruled out), given by (14).
Simulation results are reported in Table 1. Variables dated at time $t$ (i.e. $L_t, C_t, F_t, Y_t, CA_t, F_t/D_t, BT, neutralF/D, nbD, TIS$) have a single value, whereas variables dated at time $t+1$ can take a set of possible values depending on the (simulated) realisation of the shock.

### 4. RESULTS

#### 4.1 Why holding reserves?

The rationale for holding reserves, for an exporting economy which is indebted in foreign currency, originates from a structural trade off between economic growth and financial distress, as illustrated in Figure 1. A depreciation of the foreign exchange, although working as an incentive to increase the production ($G_{NP}$), can lead to falling total income growth ($Y_t$). Choosing the level of foreign reserves, $F_t$, provides additional resources (active interest and capital payback) to offset the outflow of income due to debt service ($G_{NP}$ with optimal reserves). Reserves allow the economy to reduce the dependence of the total income on the exchange rate (as in Figure 1.a) or to even reverse the sign of this relation (from negative to positive, as in Figure 1.b). The rows in Table 1 report the simulated standard deviation of total income, in percent of GDP. The sign of this sum reflects the relation between total income and exchange rate fluctuations.

Comparing optimal $F_t/D_t$ with raws $TIS$ and $neutralF/D$, numerical simulations suggest that countries with a ratio of reserves to debt around 2 are able to generate additional resources of about 2.7% of the GDP from a devaluation of 40% of the
exchange rate, and about 2% of the GDP from a devaluation of 30%, whereas the financial resources of countries close to neutral ratio (recommended by the Greenspan-Guidotti rule) are almost unaffected by shocks of the same magnitude. This consideration could explain why emerging countries have raised the ratio to values around and above 2.

The level of foreign short term debt implying no bankruptcy, $nbD$, in all numerical examples, is much lower (around 0.35% of GDP) than the average level observed for the emerging economies (6.1% of GDP, according to Rodrik, 2006). This result clearly suggests that limiting foreign capital inflow could fully eliminate the need to hold reserves to prevent financial crises.

4.2 How much to hold?

Numerical simulations reported in Table 2 suggest the following highly counterintuitive implications of the model.

- Higher values of the reserves to debt ratio ($R_z/D_z$) are associated, ceteris paribus, to lower values of the risk aversion parameter ($\sigma$).

In an exporting economy where firms are indebted to foreign creditors, both appreciation and depreciation of the exchange rate have negative effects: appreciation is associated to lower income from domestic production, whereas depreciation is associated to higher financial distress of the producers and higher bankruptcy risk. The risk aversion parameter, in this light, should be interpreted as the balance assigned by the representative agent (or, more realistically, by the authority of political economy) to these opposite sign risks. Higher short term reserves to debt ratio reflects the increasing weight assigned to the risk of firms going bankrupt with respect to the risk of economic slowdown.

- A lower risk premium charged by international lenders to domestic companies does not change the level of the investment and determines higher levels of reserves to short term debt ratio.

When the interest rate on foreign debt declines (in Table 1, $r_f - r_F$ goes from 12% to 7%), indebted firms are clearly relieved: the expected foreign debt service
falls, the bankruptcy threshold, \( \tilde{\mu}_{t+1} \), is higher and the probability of going bankrupt, \( 1 - \mathcal{G}(\tilde{\mu}_{t+1}) \), is lower, therefore, the expected cost of foreign debt for the firms decreases (FCFD). Domestic savers are also better off: the representative agent decides to increase saving by reducing current consumption (\( C_t \)); however, she purchases more reserves rather than financing a higher level of investment. This happens because the optimal level of investment is nearly insensitive to changes in \( r_d \). Figure 2 helps us to understand why. The representative firm sets the optimal level of the investment, plausibly, at a point which is close to the maximum bankruptcy threshold. Lower cost of foreign debt causes an upward shift of the function \( \tilde{\mu}_{t+1}(I_t, \ldots) \) - from the continuous to the dashed line - which implies a higher bankruptcy threshold and lower probability of going bankrupt for any level of \( I_t \), but a nearly unchanged slope of the curve around the optimal investment.

- A change in the exchange rate volatility has a negligible impact on the choice of investment, current consumption and reserves to debt ratios. Volatility, by contrast, could be a relevant determinant of the decision about whether or not to hold reserves for precautionary motives.

Comparing Panel A and Panel B of Table 1, a change in volatility has a negligible impact on the choice of investment, current consumption and reserves to debt ratios, as the marginal cost of the investment is a virtually flat curve. However, a lower value of volatility can play a crucial role in the decision of whether or not to hold foreign currency reserves. The first column in Panel A and B describe an economy which assigns a relatively high importance to the countercyclical effect of net income (\( \sigma = 4 \)) but observes a high cost of borrowing from abroad (\( r_D - r_P = 0.12 \)), thus it is on the edge between holding and not holding reserves. This economy could decide to hold foreign reserves if the exchange rate volatility is higher (Panel A) and not to hold them if volatility is lower (Panel B).

5 CONCLUDING REMARKS

This work has established that ratios well above the Greenspan-Guidotti level, derived as general equilibrium solutions, reflect the increasing weight that emerging economies have been recently assigned to the risk of firms going
bankrupt with respect to the risk of economic slowdown. As raising reserves to short term debt ratio, even being an optimal policy, implies relevant social costs for the emerging economy and is responsible for global imbalances (Rodrik, 2006; Greenwald and Stiglitz, 2010), reducing this ratio could be a desirable policy goal.

According to simulated results, to lower the ratio, we cannot rely on a better reputation of the domestic economy's private borrowers, as a lower risk premium could induce the economy to increase rather than reduce the stock of reserves. Neither does a monetary system based on lower exchange rate volatility seem to be unequivocally helpful to reduce the stock of reserves, although it could play a role in the decision about whether or not to hold reserves for precautionary motives.

The need to hold reserves to hedge against financial distress could be fully eliminated by two radical solutions. The first solution consists in reducing the level of the external short term debt to the point at which bankruptcy is not possible. This work provides insights to determine the correct level of debt where such safety is reached and suggests to control foreign capital inflow. The second solution is more obvious and consists in reforming the international monetary system to rule out the possibility of exchange rate fluctuations (examples of proposals in this direction are Mundell, 2001, D'Arista, 2004, Greenwald and Stiglitz, 2010). This solution is compatible with proposals of regional monetary unions, provided that capital inflow comes from countries of the union.
### Table 1 Numerical simulations

**Panel A – Higher exchange rate volatility**

<table>
<thead>
<tr>
<th>σ</th>
<th>0.12</th>
<th>0.07</th>
</tr>
</thead>
<tbody>
<tr>
<td>r_f-r_t</td>
<td>6.692</td>
<td>6.692</td>
</tr>
<tr>
<td>C_i</td>
<td>94.387</td>
<td>93.147</td>
</tr>
<tr>
<td>E_iC_i</td>
<td>111.130</td>
<td>112.407</td>
</tr>
<tr>
<td>F_i</td>
<td>5.021</td>
<td>6.262</td>
</tr>
<tr>
<td>S_i</td>
<td>5.613</td>
<td>6.853</td>
</tr>
<tr>
<td>C_A</td>
<td>-1.079</td>
<td>0.162</td>
</tr>
<tr>
<td>F/D</td>
<td>0.823</td>
<td>1.026</td>
</tr>
<tr>
<td>neutral F/D</td>
<td>1.055</td>
<td>1.055</td>
</tr>
<tr>
<td>nbD</td>
<td>0.338</td>
<td>0.338</td>
</tr>
<tr>
<td>1-G(μ_t+1)</td>
<td>0.041</td>
<td>0.041</td>
</tr>
<tr>
<td>EC fd</td>
<td>0.839</td>
<td>0.839</td>
</tr>
<tr>
<td>TIS</td>
<td>-1.454</td>
<td>-0.176</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C_i+1 Mean</th>
<th>Median</th>
<th>sdGDP</th>
<th>sdGDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.017</td>
<td>0.002</td>
<td>0.023</td>
<td>0.078</td>
</tr>
</tbody>
</table>

Simulated sample of μ_t, with j=1…1000: standard deviation=0.404 (k=6); mean=1.011; min value: 0.233; max value: 2.614.

**Panel B – Lower exchange rate volatility**

<table>
<thead>
<tr>
<th>σ</th>
<th>0.12</th>
<th>0.07</th>
</tr>
</thead>
<tbody>
<tr>
<td>r_f-r_t</td>
<td>6.692</td>
<td>6.692</td>
</tr>
<tr>
<td>C_i</td>
<td>99.408</td>
<td>93.147</td>
</tr>
<tr>
<td>E_iC_i</td>
<td>105.958</td>
<td>112.407</td>
</tr>
<tr>
<td>F_i</td>
<td>0</td>
<td>0.262</td>
</tr>
<tr>
<td>S_i</td>
<td>0.592</td>
<td>0.853</td>
</tr>
<tr>
<td>C_A</td>
<td>-6.1</td>
<td>0.338</td>
</tr>
<tr>
<td>F/D</td>
<td>0</td>
<td>1.026</td>
</tr>
<tr>
<td>neutral F/D</td>
<td>1.055</td>
<td>1.055</td>
</tr>
<tr>
<td>nbD</td>
<td>0.338</td>
<td>0.338</td>
</tr>
<tr>
<td>1-G(μ_t+1)</td>
<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td>EC fd</td>
<td>0.879</td>
<td>0.879</td>
</tr>
<tr>
<td>TIS</td>
<td>-6.626</td>
<td>-0.176</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C_i+1 Mean</th>
<th>Median</th>
<th>sdGDP</th>
<th>sdGDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.015</td>
<td>0.015</td>
<td>0.015</td>
<td>0.015</td>
</tr>
</tbody>
</table>

Simulated sample of μ_t, with j=1…1000: standard deviation=0.318 (k=10); mean=1.012; min value: 0.330; max value: 2.119

**Simulation parameters.** Normalisation: ε = 1, Y = 100. Investment function: δ = 2, β = 0.7, w = 4, a = 0.03. Interest rate on foreign reserves: r_f = 0.03. Intertemporal rate of substitution: 1/β = 1/0.03. Private debt to foreign creditors: D = 6.1. Bankruptcy repayment: b = 2.08.
Figure-1: The hedging effect of holding reserves

(1.a) GDP and GNP when $\sigma=3$

(1.b) GDP and GNP when $\sigma=1$

Figure-2: Effect of change in the interest rate gap on firm’s equilibrium conditions

(2.a) Bankruptcy threshold

(2.b) Firm’s equilibrium solution
BIBLIOGRAPHY


