

## CRUDE OIL PRICE MODELLING WITH LEVY PROCESS

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### —Abstract —

The increased oil prices worldwide are having a great impact on all economic activities. That's why research on the dynamic behavior of crude oil prices has become a hot issue in recent years. Especially the recent changes in crude oil price behaviour between 2007 and 2009 revived the question about the underlying dynamics governing crude oil prices. To understand the behavior of the oil market there is a need to understand the stochastic models of oil prices. Their dynamics were characterized by high volatility, high intensity jumps, and strong upward drift, indicating that oil markets were constantly out-of-equilibrium. The aim of this study is to model oil price returns by Lévy process including the temporal, spectral and distributional properties of the data set. Our findings could be helpful for monitoring oil markets and we expect that the analysis presented in this paper is useful for researchers and energy economists interested in predicting crude oil price and return.

**Key Words:** *Oil, Stochastic Modelling, Levy Process, ARMA, GARCH, COGARCH*

**JEL Classification:** C01, C51

## 1. INTRODUCTION

Crude oil is a commodity of great importance which is closely watched by policy-makers, producers, consumers and financial market participants and its price is one of the most global economic indicators in many economies.

The basic time series models are based on the assumption of normality of asset returns. However, many empirical studies show that the assumption of normality is a poor approximation in the real world because returns have some significant features such as jumps, semi-heavy tails and asymmetry. In traditional diffusion models, price movements are very small in short period of time. But in real markets, prices may be show big jumps in short time periods. For these reasons, diffusion models used in finance is not a sufficient model. A good model must be allow for discontinuities and jumps in price process. At this point, continuous time models would have many advantages with respect to discrete ones and Lévy processes can be a valuable tool in stochastic modeling.

Over the last two decades different models have been proposed to justify the stochastic behavior of oil prices. Cortazar and Schwartz (2002) three-factor modeling of oil prices considers the price behavior under the risk neutral measure, as well as the volatility term structure model of returns. Brunetti (1999) highlights the long-memory effect in commodity prices.

In Krichene's IMF's Working Paper (2006), it is stated that, oil price distribution is left-skewed, implying that downward jumps of smaller size were more frequent than upward jumps of larger size; as the mean is positive and high, smaller jumps are outweighed by larger jumps. The distribution has also fat tails, meaning that large jumps tended to occur more frequently than in the normal case. These empirical findings about oil prices were typical of financial time series as noted in Clark (1973), Fama (1965), and Mandelbrot (1963). These facts suggested modeling the oil price process as a jump-diffusion or, in a more general way, as a Levy process (Cont and Tankov, 2004).

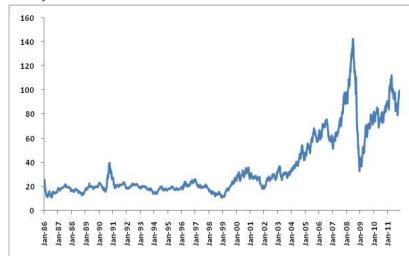
In this study, continuous time GARCH model, namely COGARCH (Klüppelberg, 2004:3) is used for the crude oil price modeling.

## 2. TIME SERIES ANALYSIS

### 2.1. Analysis in Time Domain

The data on figure 1 is the West Texas Intermediate (WTI) crude oil spot prices (in US dollars per barrel). The data set consist of weekly closing prices over the period from January 3, 1986 to December 16, 2011 and contains 1355 observations for WTI crude oil markets.

Figure 1: Crude oil weekly spot prices over the period from Jan 3, 1986 to Dec 16, 2011



Autocorrelation values decrease slowly and Partial Autocorrelation values sharply converge to almost zero level. The decrease in autocorrelation could be thought as the fact that random shocks to the system dissipate with time. It could be concluded that the Crude Oil (WTI) prices have a trend and so are mean non-stationary. Finally, considering the slow decrease in autocorrelation values, it could be concluded that there is a long memory structure in the data.

To achieve mean stationary logarithmic difference of the series is calculated and it is found that the logarithmic return of oil price series seems to have no trend. The next step is to check that whether it is a non-stationary or not. High levels of Autocorrelation indicates MA, and high levels of Partial Autocorrelation indicate AR.



Figure 3: Autocorrelation and Partial Autocorrelation of logarithmic spot prices

Hannan-Rissanen procedure of estimating ARMA parameters will be applied to obtain the AR and MA parameters. Specifically, the Hannan-Rissanen procedure consists of following steps.

1. Use the Levinson-Durbin algorithm to fit models AR(i) to the data.
2. Calculate the AIC values of the fitted AR models, and choose the model AR(k) that yields the smallest AIC value.
3. Calculate the residuals from the fitted AR(k) model.
4. Estimate ARMA(p,q) coefficients using the least squares method.
5. Select the model that has the lowest BIC value.

When Hannan-Rissanen estimate procedure has been applied to obtain the best model, the first best three models are obtained as follows:

1-MA [8]

[{0.1131, -0.0664, 0.0916, 0.0459, -0.0098, -0.0440, -0.0107, 0.0919}, 0.0018]

2-ARMA [1,8]

[{0.2352,0.03245,0.2302,0.04077,0.2949,0.1304},0.0006]

3-MA [9]

[{0.1126, -0.0665, 0.0916, 0.0461, -0.0091 -0.0442, -0.0103, 0.0923, 0.0214}, 0.00184855]

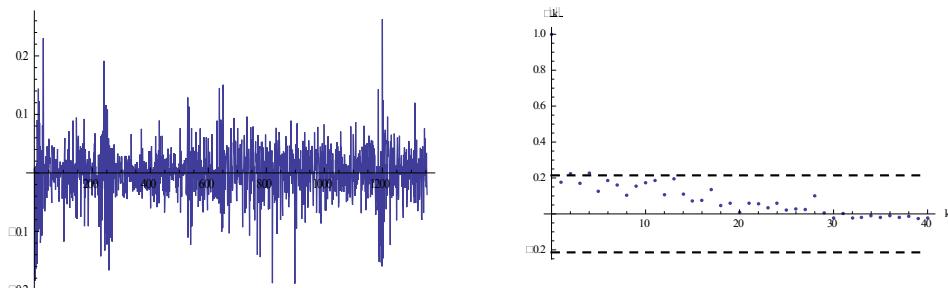
The next step is to check the related BIC and AIC values to control if the best model has the lowest AIC and BIC values. The results at below shows that MA[8] is the best model.

BIC:{-6.026, -5.994, -5.992} AIC:{-6.186, -6.175, -6.173}

If we examine the residuals we found that they behave like a white noise process with zero mean and constant variance. So the behavior of the residuals is used to test the adequacy of the fitted model.

The first step in residual testing is to calculate the residuals given the fitted model and observed data. The below graph is representing the residuals calculated and it gives no indication of deviation from stationary random noise. Also, autocorrelation function states that the residuals are white noise.

Figure 4: Residuals and autocorrelations of MA[8] Model



In addition to these tests, Portmanteau Test can also be executed to check that the first  $h$  correlations of residuals together have any significant autocorrelation or not. The test statistic is calculated as

$$Q_{1,h} = n(n+2)\sum_{k=1}^h (k-1)^2 \hat{\rho}_k^2 - [n(n-1)^2 / ((n-h))], \quad (1)$$

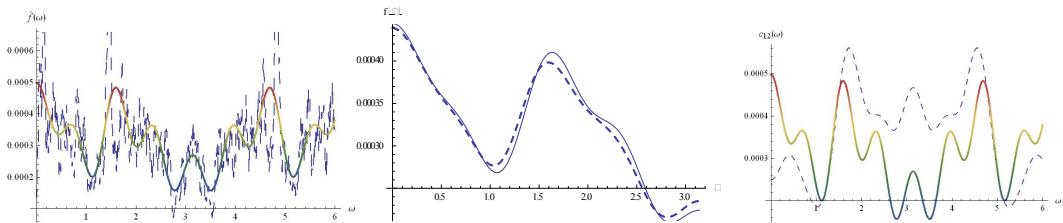
where  $n$  is the number of data points and  $h$  is the number of autocorrelations we want to evaluate. The portmanteau statistic calculated for MA(8) for the first 35 autocorrelations is 41.17 while the critical value (with a degree of freedom equal to  $h-p-q=35-1-0=34$ )  $\chi^2_{0.95}(34) = 49.59$ . Thus, the portmanteau statistics also indicates that we do not have enough evidence to believe there is autocorrelation left in the residuals for MA(8) model.

## 2.2. Analysis in Frequency Domain

We have so far studied the time series analysis in the time domain. Another approach is to analyze the time series in Fourier space or in the frequency domain. The techniques used in the frequency domain fall under the general rubric of spectral analysis and the fundamental tool is the Fourier transform.

The spectrum of a stationary time series is the counterpart of a covariance function in frequency domain. That is, it is the Fourier transform of the covariance function. In the spectrum analysis the aim is to check whether the data fits to the spectrum of MA(8) model.

Figure 5: Smoothed Spectrum of Logarithmic Return Data vs Model (MA[8])



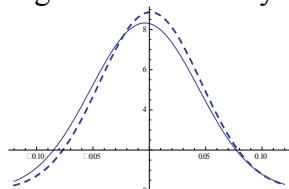
## Spectrum, Hanning and Barlet Windows

The spectrum analysis can also be carried out for both Hanning and Barlet Windows and it is found that the results are also quite in line each other. In other words, the model spectrum fits to data spectrum in both cases. The related graphs are given below:

Figure 6:

In this step, the probability density functions of both original logarithmic return data and the series produced from the first model, which is MA[8], are graphed by first generating the series from the model and then calculating the mean and standard deviations.

Figure 7: Probability Distribution Functions of Log Returns and Model Series



Then, the kurtosis and skewness of residuals have been calculated. They have been found as 6.1958 and -0.2143 respectively.

The data has to be also checked if there is any GARCH effect of not. ARCH LM test is used for this purpose and the result shows that there is GARCH effect in the series which is to be modeled. (LM Statistic : 32.3812 & 95% Quantile : 3.84146)

GARCH effect is analyzed through E-Views and it is found that GARCH[1,1] is the best fit via lowest AIC value. The model parameters are as follows:

GARCH Constant : 8.13E-05, Residual- $\epsilon^2$  : 0.092191, GARCH-1 : 0.860032

So, the model becomes;

$$r_t = 0.1131 \quad \epsilon_t - 0.0664 \quad \epsilon_{t-1} + 0.0916 \quad \epsilon_{t-2} - 0.0459 \quad \epsilon_{t-3} - 0.0098 \quad \epsilon_{t-4} \\ - 0.0440 \quad \epsilon_{t-5} - 0.0107 \quad \epsilon_{t-6} + 0.0919 \quad \epsilon_{t-7} + 0.0018 \quad r_{t-8}$$

$$\sigma_t^2 = 8.15E-05 + 0.091308 a_{t-1}^2 + 0.862718 \sigma_{t-1}^2$$

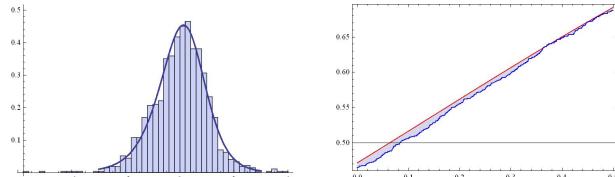
To be able to fit the best distribution to GARCH Model error series, Normal, Cauchy, Student-t, Weibull, Gumbel and Johnson SU distributions are being

applied and it is found that the best fitted one is the Johnson Su Distribution. The distribution parameters are as follows;

$$\mathbf{a} = 0.3936, \mathbf{b} = 1.9668, \mathbf{c} = 0.4064, \mathbf{d} = 1.7032$$

Regarding the histogram and cumulative distribution functions Johnson Su Distribution seems to be the best distribution to GARCH Model error series.

Figure 11: Histogram, and Cumulative Distribution Function of GARCH Errors



The statistic and the related p-values are presented in the table at below.

Table 1: Goodness of Fit test results for Johnson SU

	Statistic	P-Value
Anderson-Darling	3.75672	$6.37774 \cdot 10^{-7}$
Cramér-von Mises	0.614592	0.
Jarque-Bera ALM	251.489	0.
Kolmogorov-Smirnov	0.0388138	0.0000336107
Kuiper	0.0734928	$7.15575 \cdot 10^{-7}$
Pearson $\chi^2$	63.6662	0.00105645
Shapiro-Wilk	0.981098	$2.52113 \cdot 10^{-12}$
Watson $U^2$	0.56362	0.000133572

The test results for the rest are also quite satisfactory. This means that the results at above table confirm that Johnson Su Distribution is quite good fit for the GARCH error terms.

### 3. CONTINUOUS MODELLING

#### 3.1. COGARCH(1,1) process

In this part COGARCH is used in terms of continuous modeling.

COGARCH(1,1) process is defined as the solution to the following stochastic differential equations

$$(2)$$

$$d\sigma_t^2 = (\beta - \eta\sigma_t^2)dt + \varphi\sigma_t^2 d[L, L]_t^{(a)}, t \geq 0$$

$$(3)$$

$$d\sigma_t^2 = (\beta - \eta\sigma_t^2)dt + \varphi\sigma_t^2 d[L, L]_t^{(a)}, t \geq 0$$

Where  $d[L, L]_t^{(d)} = (\Delta L_t)^2$  and  $\Delta L_t = L_t - L_{t-1}$

Assuming the data consist of intervals of time length equal to  $r$ , the equations 2 and 3 become

$$G_t^{(r)} = G_t - G_{t-r} = \int_{[t-r,t]} \sigma_s dL_s \quad (4)$$

$$\sigma_{rn}^{2(r)} = \sigma_{rn}^{2(r)} - \sigma_{r(n-1)}^{2(r)} = \beta r - \eta \int_{[r(n-1),rn]} \sigma_s^2 ds + \varphi \int_{[r(n-1),rn]} \sigma_s^2 d[L, L]_s^{(d)} \quad (5)$$

For  $r = 1$ ;

$$\sigma_n^2 = \sigma_{n-1}^2 + \beta - \eta \int_{[n-1,n]} \sigma_s^2 ds + \varphi \sum_{n-1 < s \leq n} \sigma_s^2 (\Delta L_s)^2 \quad (6)$$

We approximate the integral and the sum on the right hand side of the equation. For the integral; we use a simple Euler Approximation

$$\int_{[n-1,n]} \sigma_s^2 ds \approx \sigma_{n-1}^2$$

We can also approximate the sum as;

$$\sum_{n-1 < s \leq n} \sigma_s^2 (\Delta L_s)^2 = (G_n - G_{n-1})^2 = (G_n^{(1)})^2$$

Thus we end up with an discretized version of equation 3

$$\sigma_n^2 = \beta + (1 - \eta) \sigma_{n-1}^2 + \varphi (G_n^{(1)})^2 \quad (7)$$

We can clearly see the analogue with the Garch(1,1) which is;

$$\sigma_n^2 = a + b \sigma_{n-1}^2 + c(Y_i)^2 \quad (8)$$

where  $Y_i = \sigma_{i-1} \varepsilon_i$

### 3.2. Application

Continuous time model is defined by Klüpperberg as follows;

$$G_{i,n} = G_{i-1,n} + \sigma_{i-1,n} \sqrt{\Delta t_i(n)} \varepsilon_{i,n} \quad (9)$$

$$\sigma_{i,n}^2 = \beta \Delta t_i(n) + (1 + \varphi \Delta t_i(n) \varepsilon_{i,n}^2) e^{-\eta \Delta t_i(n)} \sigma_{i-1,n}^2 \quad (10)$$

With some simplifications;

$$\sigma_{i,n}^2 = \beta \Delta t_i(n) + e^{-\eta \Delta t_i(n)} \sigma_{i-1,n}^2 + \varphi \Delta t_i(n) e^{-\eta \Delta t_i(n)} \sigma_{i-1,n}^2 \varepsilon_{i,n}^2 \quad (11)$$

since the Garch(1,1) can be written as

$$\sigma_n^2 = \beta + \delta \sigma_{n-1}^2 + \lambda (Y_i)^2 \quad (12)$$

Then, with equation 10 and 11 we can find the parameters for the COGARCH(1) as follows

$$\beta = \beta, \quad e^{-\eta} = \delta \Rightarrow \eta = -\ln(\delta), \quad \varphi e^{-\eta} = \lambda \Rightarrow \varphi \delta = \lambda \Rightarrow \varphi = \frac{\lambda}{\delta}$$

When we disregard the MA term from the discrete model the model reads as:  
 $r_t = 0.001515 + a_t$

$$\sigma_t^2 = 0.0000815 + 0.091308 a_{t-1}^2 + 0.862718 \sigma_{t-1}^2$$

After estimating an appropriate discrete-time GARCH process, the next step in the analysis would be estimating a continuous-time model by using discrete model parameters. The COGARCH(1,1) model which takes the form of

(13)

$$d\sigma_t^2 = (\beta - \eta \sigma_t^2) dt + \varphi \sigma_t^2 d[L, L]_t^{(d)}, t \geq 0 \quad (14)$$

Where  $d[L, L]_t^{(d)} = (\Delta L_t)^2$  and  $\Delta L_t = L_t - L_{t-1}$

The parameters of the continuous model are equal to

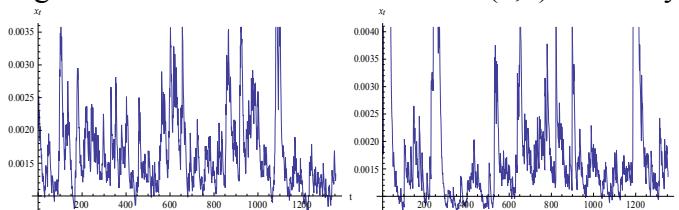
$$\beta = \beta, \eta = -\ln(\delta), \varphi = \lambda / \delta$$

The COGARCH(1,1) model parameters in this case are

$$\beta = 0.8.15e-5 \quad \eta = -\ln(0.8627) = 0.1476 \quad \varphi = \lambda / \delta = 0.0913 / 0.8627 = 0.1058$$

In the simulation process we use numerical solutions for  $G_t$  and  $\sigma_t^2$  and we use Lévy process driven by Johnson SU process. Figures at below show the time plot of the simulated values of COGARCH(1,1) process, the volatility processes generated by GARCH(1,1) which are compared the to real volatility.

Figure 12: COGARCH and GARCH (1,1) Volatility



### 3. CONCLUSION

Logarithmic return of crude oil weekly closing prices was modeled with the best candidate model MA(8)~GARCH(1,1). In order to run continuous model we dropped the MA(8) term and by using the parameters from the discrete model, continuous model COGARCH(1,1) was applied to the data. Volatility of simulated data from discrete and continuous models compared with the real data volatility. We showed that the simulated GARCH volatility and COGARCH volatility appears to follow the same pattern of jumps. Furthermore, both models imitate the real return data volatility.

## BIBLIOGRAPHY

- Applebaum, D. (2011), Lecture given at Koç University on Levy Process
- Brunett, Celso, 1999, “Long Memory, The Taylor Effect and Intraday Volatility in Commodity Futures Markets
- Barndorff-Nielsen, Ole E., Neil Shephard (2001), “Modelling by Lévy Processes for Financial Economics”, Birkhauser:Boston, pp.283-318.
- Clark, P. K., 1973, “A Subordinated Stochastic Process with Finite Variance for Speculative Prices,” *Econometrica*, Vol. 41, pp. 135–155.
- Cont, R. and Tankov, P., 2004, *Financial Modeling with Jump Processes*, (Chapman&Hall/CRC).
- Cortazara, G. and Schwartzb, E., 2003, “*Implementing a Stochastic Model for Oil Futures Prices*,”
- Fama, E.F., 1965, “The Behavior of Stock Market Prices,” *Journal of Business*, Vol. 34, 420–429.
- Hannan, E. J., Rissanen J. (1982), “Recursive Estimation of Mixed Autoregressive-Moving Average Order”, *Biometrika*, Vol. 69, No. 1, pp. 81-94
- Krichene, Noureddine, 2006 “Recent Dynamics of Crude Oil Prices,” *IMF Working Paper*
- Klüppelberg, Claudia, Alexander Lindner and Ross Maller (2004), “A Continuous Time GARCH Process Driven by a Lévy Process:Stationary and Second Order Behaviour”, *Journal of Applied Probability*, Vol. 41, pp.601-622
- Müller, G., Durand, R., Maller, R., Klüppelberg, C., “Analysis of stock market volatility by continuous-time GARCH models”, Stock Market Volatility, Chapman Hall/Taylor and Francis, London, pp. 31 50, 2009